Electron Density in the Extended Corona — Two Views

A. L. Berman
TDA Engineering Office

Recent analyses of Viking and Mariner solar conjunction radio metric data have led to two significantly different views of the average radial dependence of electron density in the extended corona ($5r_{\odot} \leqslant r \leqslant 1$ AU):

$$N_e(r) \propto r^{-2}$$

and

$$N_e(r) \propto r^{-2.3}$$

This article compares the two models and concludes that the "steeper" model ($r^{-2.3}$): (1) is in excellent agreement with other experimental observations of coronal electron density, (2) is consistent with the predicted and observed radial dependence of the solar wind velocity, and (3) augments the case for a turbulence scale that expands linearly with radial distance, when considered in combination with recent observations of the radial dependence of RMS phase fluctuations.

I. Introduction

Recent analyses of Viking and Mariner radio metric data acquired during solar conjunction have led to two significantly differing views of the equatorial coronal electron density function. In a series of recent articles (e.g., Refs. 1-3) Berman, using Viking S-band doppler noise, has shown that the radial dependence of RMS phase (ϕ) in the extended corona[1] is:

$$\phi \propto a^{-1.30}$$

where a = signal path closest approach point, and emphasizes that this result can be explained as the signal path integration of the following nominal electron density (N_e) function:

$$N_e(r) \propto r^{-2.3}$$

where r = heliocentric distance.

More recently, Callahan (Ref. 4), analyzing Viking S–X doppler, has also concluded that phase fluctuations are of the form:

$$\phi \propto a^{-1.3}$$

[1] Here to be considered as approximately $5r_{\odot} \lesssim r \lesssim 1$ AU, where r = radial distance and $r_{\odot}$ = solar radius.

However, Callahan infers from the above relationship the following electron density fluctuation (n) radial dependence

$$n \propto r^{-1.8}$$

A very common assumption made in coronal investigations is:

$$n/N_e = \epsilon; \epsilon \neq \epsilon (r)$$

Thus implying (for the Callahan Inference)

$$N_e(r) \propto r^{-1.8}$$

Muhleman (Ref. 5), analyzing Mariner 6 and Mariner 7 S-band range data, has recently reported the following electron density models:

$$N_e(r) \propto r^{-2.05} \text{ (Mariner 6)}$$

$$N_e(r) \propto r^{-2.08} \text{ (Mariner 7)}$$

Although the Muhleman and Callahan results are not completely complementary (in terms of a radially constant ratio $\epsilon = n/N_e$), they do provide a composite picture of a significantly less "steep" corona in the way of radial dependence.

That the difference between

$$N_e(r) \propto r^{-2}$$

and

$$N_e(r) \propto r^{-2.3}$$

is substantial is easily seen by assuming the commonly accepted average in situ measured value of approximately 7.5 electrons/cm^3 (Refs. 6 and 7) and extrapolating back to 5 solar radii (5 $r_\odot$). One has

Radial dependence	N_e @ 1 AU, electrons/cm^3	N_e @ 5 $r_\odot$, electrons/cm^3
r^{-2}	7.5	13,900
$r^{-2.3}$	7.5	42,900

so that the difference in the models at 5 $r_\odot$ is seen to be a factor of approximately 3. It will thus be the purpose of the following sections to: (1) explore the theoretical basis of the phase fluctuation – electron density relationship, and (2) ascertain whether one or the other of the two proposed models for N_e is more consistent with the many other experimental observations of the corona made over the last decade or so.

II. The Phase Fluctuation—Electron Density Fluctuation Relationship

In 1968 Hollweg (Ref. 8), using a statistical ray analysis based on geometric optics similar to that originally formulated by Chandrasekhar in 1952 (Ref. 9), derived the following expression for RMS phase induced by electron density fluctuations in the solar corona:

$$\phi^2 \simeq 2\sqrt{\pi}\, r_e^2\, \lambda^2 \epsilon^2 \int_a^\infty [N_e(r)]^2\, L_t(r) \frac{rdr}{\sqrt{r^2 - a^2}}$$

where

λ = signal wavelength
r_e = classical electron radius
r = radial distance
a = signal closest approach point
ϵ = fluctuation to density ratio
L_t = *transverse* scale of fluctuations
N_e = electron density

Functionally similar expressions are given in Jokipii, (1969, Ref. 10), and Little (1970, Ref. 11). Little notes that this expression is valid for different functional forms of the fluctuation spectrum, with only a slight change in the numerical factor. Hollweg (1970, Ref. 12), subsequently derives the expression *specifically* for the (now commonly accepted) power law fluctuation spectrum, as follows:

$$\phi^2 \simeq 3\pi \left(\frac{\alpha - 1}{\alpha}\right) r_e^2\, \lambda^2 \epsilon^2 \int_a^\infty [N_e(r)]^2\, L(r) \frac{rdr}{\sqrt{r^2 - a^2}}$$

with

$\alpha + 2$ = exponent of the three dimensional spatial spectrum $\simeq 3.5$
L = outer scale of turbulence

Hollweg (Ref. 13, 1968) considers the relationship:

$$L_t(r) \propto r$$

to be a result of inhomogeneities expanding with a radially out-flowing solar wind. In Ref. 8, Hollweg treats the most common assumptions of a constant transverse scale and one linear with radial distance:

$$L_t(r) = 200 \text{ km}$$

and

$$L_t(r) = 30(r/r_\odot)\text{km}$$

where

$$r_\odot = \text{solar radius}$$

Substitution of these assumptions for the transverse scale produces the following results[2] (with $N_e \propto r^{-(2+\xi)}$ and $n/N_e = \epsilon$):

(1) Scale constant with radial distance

$$\phi^2 \propto \int_a^\infty \frac{1}{(r^{2+\xi})^2} \frac{rdr}{\sqrt{r^2 - a^2}} \propto (a^{-(1.5+\xi)})^2$$

(2) Scale linear with radial distance

$$\phi^2 \propto \int_a^\infty \frac{1}{(r^{2+\xi})^2} \frac{r^2 dr}{\sqrt{r^2 - a^2}} \propto (a^{-(1.0+\xi)})^2$$

[2]The dependence on closest approach distance (a) is obtained by noting that the integral:

$$\int_a^\infty \frac{r^\alpha dr}{\sqrt{r^2 - a^2}}$$

is transformed via the substitution $r = a (\cos x)^{-1}$ to

$$a^\alpha \int_0^{\pi/2} (\cos x)^{-(1+\alpha)} dx$$

It is thus seen that the constant scale produces the relationship between RMS phase and electron density fluctuations inferred by Callahan (as described in Sect. I), and similarly, usage of the linear transverse scale produces the functional relationship argued by Berman.

The case for a linear scale is made by Little (1970, Ref. 11), and Houminer (1973, Ref. 14), among others. Their data in support of a linear scale is reproduced here in Figs. 1 and 2. More recently, Jokipii (1973, Ref. 15), and Woo (1977, Ref. 16), among others, have considered a constant correlation scale on the order of:

$$L \sim 2 \times 10^6 \text{ to } 1 \times 10^7 \text{km}$$

Scales on this order are obtained from spacecraft measured correlation times (τ_c) at approximately 1 AU of:

$$\tau_c \sim 6 \times 10^3 \text{ to } 3 \times 10^4 \text{s}$$

with

$$L \sim v_r \times \tau_c$$

where

$$v_r = \text{solar wind velocity } (\sim 350 \text{ km/s})$$

What is thus obtained is a *radial* correlation length at approximately 1 AU; more appropriate for usage with a columnar phase measurement such as doppler noise would be a linear *transverse* correlation length as described by Hollweg and experimentally observed by Little and Houminer.

Using Viking S-Band doppler noise and near simultaneous Viking S-X range data, Berman (1977, Ref. 3), derived the following scale for 60-s sample interval doppler noise (time scale of the observations ~ 15 × 60 s):

$$L(a) = \left[\frac{0.43}{\epsilon^2}\right] (a/r_\odot), \text{ km}$$

or, assuming nominal bounding values of ϵ:

$$L(a) = 43(a/r_\odot), \text{ km}; \ \epsilon = 0.1$$

$$L(a) = 4300(a/r_\odot), \text{ km}; \ \epsilon = 0.01$$

Factoring into the scale the fluctuation frequency (ν) dependence (Berman, 1977, Ref. 17), one has:

$$L(a) = \frac{0.43}{\epsilon^2}\left(\frac{a}{r_\odot}\right)\left(\frac{\tau}{60}\right)^{1.4}, \text{km}$$

where

τ = doppler sample interval, s

$\nu \simeq (2 \times 15 \times \tau)^{-1}$

It is thus seen (i.e., given the *well determined* radial dependence of phase fluctuations and the Hollweg derived relationship) that if other experimental observations of the radial dependence of electron density support a $r^{-2.3}$ corona, the case for a linear transverse scale is considerably strengthened.

III. Experimental Observations of Electron Density in the Extended Corona

Over the last decade, a sizeable number of experiments, utilizing a variety of techniques, have been performed to measure and determine electron density in the extended corona. Table 1 is a comprehensive listing of (the partial results of) these experiments. Assuming an electron density of the form:

$$N_e(r) \propto r^{-(2+\xi)}$$

the table presents the determined (or calculated) values of ξ. Of the thirteen values listed, the mean value is:

$$\bar{\xi} = 0.298$$

with a one standard deviation of

$$1\sigma(\xi) = 0.17$$

These results would certainly appear to argue strongly for an average corona of:

$$N_e(r) \propto r^{-2.3}$$

Many of these determinations of electron density utilized data whose closest approach points were in a region << 1 AU. A slightly different method of proceeding would be to select electron density values from some "interior" region approximating the region of observations, and then apply the powerful boundary condition of the *average* electron density at 1 AU:

$$N_e\,(215\, r_\odot) \approx 7.5 \text{ electrons/cm}^3$$

which is (reasonably) well known from in situ spacecraft measurements. In this regard, the following electron density values at 10 $r_\odot$ are selected:

Newkirk (1967, Ref. 24; a compilation of techniques	9800 electrons/cm^3
Saito (1970, Ref. 23; photometry and coronameter measurements with heliographic latitude solved for)	8100 electrons/cm^3
Counselman (1972, Ref. 20; pulsar data with heliographic latitude solved for)	8400 electrons/cm^3
Weisberg (1976, Ref. 19; pulsar data with heliographic latitude solved for)	8000 electrons/cm^3

The above yield an average electron density at 10 $r_\odot$ of:

$$N_e(10\, r_\odot) = 8575 \text{ electrons/cm}^3$$

It is encouraging that these diverse values for $r = 10\, r_\odot$ are nicely clustered about the mean value. Solving for a corona of the form:

$$N_e \propto r^{-(2+\xi)}$$

where

$$N_e\,(10\, r_\odot) = 8575 \text{ electrons/cm}^3$$

$$N_e\,(215\, r_\odot) = 7.5 \text{ electrons/cm}^3$$

yields:

$$\xi = 0.30$$

and

$$N_e(r) \propto r^{-2.3}$$

or, a similar value to the average computed from Table 1.

IV. Relationship Between Solar Wind Velocity and Density

One writes the condition for constant mass efflux (Hollweg, 1968, Ref. 13), as:

$$F = N_e(r) v_r(r) r^2$$

where

F = constant

$v_r(r)$ = radial component of solar wind velocity

Hence, one might expect:

$$N_e = \frac{F}{r^2 v_r(r)}$$

Now at 1 AU, the average solar wind velocity is reasonably well known (Hundhausen, 1972, Ref. 26; several years of Vela spacecraft data):

$$v_r(215\ r_\odot) \approx 400 \text{ km/s}$$

At $r = 10\ r_\odot$, one can use values from Models[3] of Hartle and Barnes (Ref. 26), Wolff, Brandt, and Southwick (Ref. 26), and Brandt, Wolff, and Cassinelli (Ref. 27), as follows:

$v_r(10\ r_\odot) \sim 170$ km/s (Hartle and Barnes)

$v_r(10\ r_\odot) \sim 185$ km/s (Wolff, Brandt, and Southwick)

$v_r(10\ r_\odot) \sim 160$ km/s (Brandt, Wolff, and Cassinelli)

or, an average value at $r = 10\ r_\odot$ of:

$$v_r(10\ r_\odot) \sim 172 \text{ km/s}$$

Assuming a power law[4] model for the solar wind velocity and solving for the resultant *radial dependence* of the solar wind velocity, one has:

$$v_r(r) \propto r^{0.28}$$

hence, the condition of constant mass efflux predicts:

$$N_e(r) = F \frac{1}{r^2 v_r(r)}$$

$$= F \frac{1}{r^2 r^{.28}}$$

$$\propto r^{-2.28}$$

or, once again, the familiar value.

One thus notes that for a coronal electron density of the form

$$N_e \propto r^{-(2+\xi)}$$

the parameter ξ can be simply identified as the radial dependence of the solar wind velocity.

V. Comparison of RMS Phase to the Scintillation Index

Using dual frequency Pioneer 9 spacecraft data, H. Chang (1976, Ref. 29), was able to make simultaneous observations of the integrated electron density (I) and the scintillation index[5] (m). He found the scintillation index to be proportional to the integrated electron density, $m \propto I$.

Both the scintillation index and RMS phase are derived from the integrated temporal columnar fluctuation spectrum (P):

$$m^2(a) \sim \int P(a, \nu) d\nu$$

$$\phi^2(a) \sim \int P(a, \nu) d\nu$$

[3] The various models depicting an increasing solar wind velocity with radial distance are substantiated by experimental observations (e.g., Ekers, 1970, Ref. 28).

[4] It is recognized that a power law assumption for the solar wind velocity in the extended corona is only approximate, similar to the power law assumption for density in the extended corona.

[5] A measure of received signal level variations induced by electron density fluctuations along the signal path.

Hence, Chang's data should certainly imply that RMS phase is also proportional to integrated electron density. The condition of $\phi(a) \propto I$ combined with

$$n/N_e = \epsilon;\ \epsilon \neq \epsilon(r)$$

requires (as shown in Sect. II):

$$L_t(r) \propto r$$

Thus, Chang's findings additionally substantiate the case for a linear transverse scale.

VI. Conclusions

It is here concluded that $N_e(r) \propto r^{-2.3}$ is a very reasonable assumption for the average radial dependence of electron density in the extended corona, based on the very favorable comparisons to:

(1) Other experimental observations of the radial dependence of electron density.

(2) The predicted and observed behavior of the solar wind velocity.

(3) The observed relationship between the scintillation index and integrated electron density.

Accepting this conclusion, the following observations and assumptions form a self-consistent set in the region $5r_\odot \leqslant r \leqslant 1$ AU:

$$\phi(a) \propto a^{-1.3}$$

$$N_e(r) \propto r^{-2.3}$$

$$L_t(r) \propto r$$

$$n(r) \propto r^{-2.3}$$

$$n/N_e = \epsilon;\ \epsilon \neq \epsilon(r)$$

$$v_r(r) \propto r^{0.3}$$

On the other hand, if one combines the Callahan inference with the *least* steep corona experimentally reported (Muhleman; Mariner 6, 1977) and the radial dependence of RMS phase:

$$\phi(a) \propto a^{-1.3}$$

$$n(r) \propto r^{-1.8}$$

$$N_e(r) \propto r^{-2.05}$$

then it is required that

$$L(r) \propto L_0$$

$$n/N_e = \epsilon(r)$$

$$\propto r^{0.25}$$

$$v_r(r) \propto r^{0.05}$$

In regard to these required conditions, it is difficult to accept that the turbulence per unit density (ϵ) increases with radial distance; even more difficult to reconcile is the required near constancy ($\propto r^{0.05}$) of the solar wind velocity for $5\,r_\odot \leqslant r \leqslant 1$ AU, in contradiction to the predicted and observed *average* radial dependence ($\propto r^{0.3}$) of the solar wind velocity in this region.

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Table 1. Electron density measurements in the solar corona

Source	Reference	Year	ξ	Type of measurement
Edenhofer	18	1977	0.2	S-band range, Helios
Berman	2	1977	0.30	S-band doppler noise, Viking
Muhleman	5	1977	0.05	S-band range, Mariner 6
Muhleman	5	1977	0.08	S-band range, Mariner 7
Weisberg	19	1976	0.3[a]	Pulsar time delay
Counselman	20	1972	0.4[b]	Pulsar time delay
Muhleman	21	1971	0.41	S-band range, Mariner 6
Muhleman	22	1970	0.33	Radio interferometry
Saito	23	1970	0.5	Photometry
Newkirk	24	1967	0.34[c]	Compilation of techniques
Blackwell	25	1967	0.33	Solar eclipse
Blackwell	25	1967	0.33	Solar eclipse
Blackwell	25	1966	0.3	Solar eclipse

[a]One of several solutions; this solution in best agreement with average in situ density values at 1 AU.

[b]One of several solutions; this solution included heliographic latitude.

[c]Computed between $N_e(10r_\odot)$ and average in situ value (7.5 electrons/cm^3) at 1 AU.

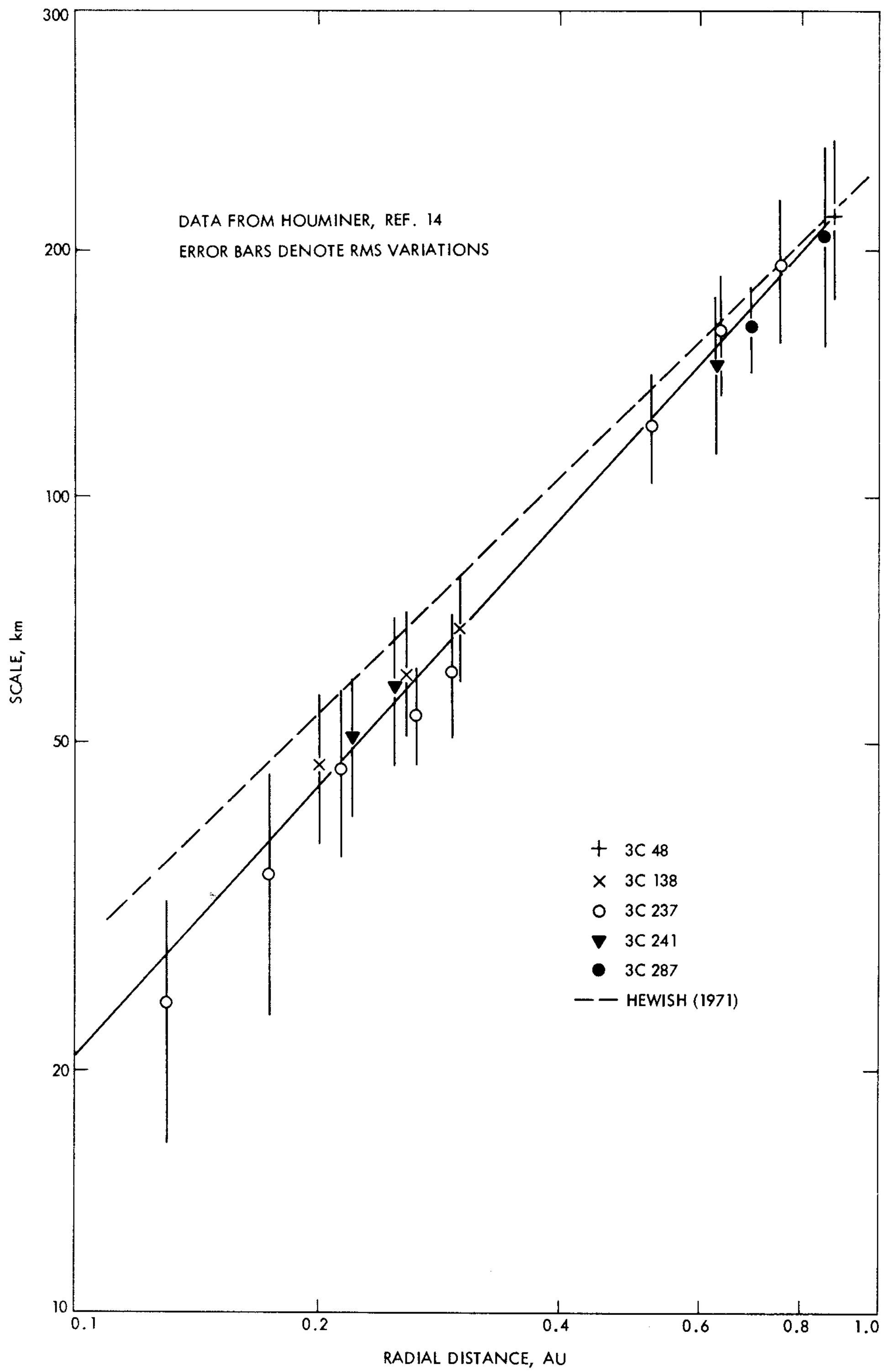

Fig. 1. Scale size of electron density irregularities at heliocentric distances between 0.1 AU < r < 1.0 AU

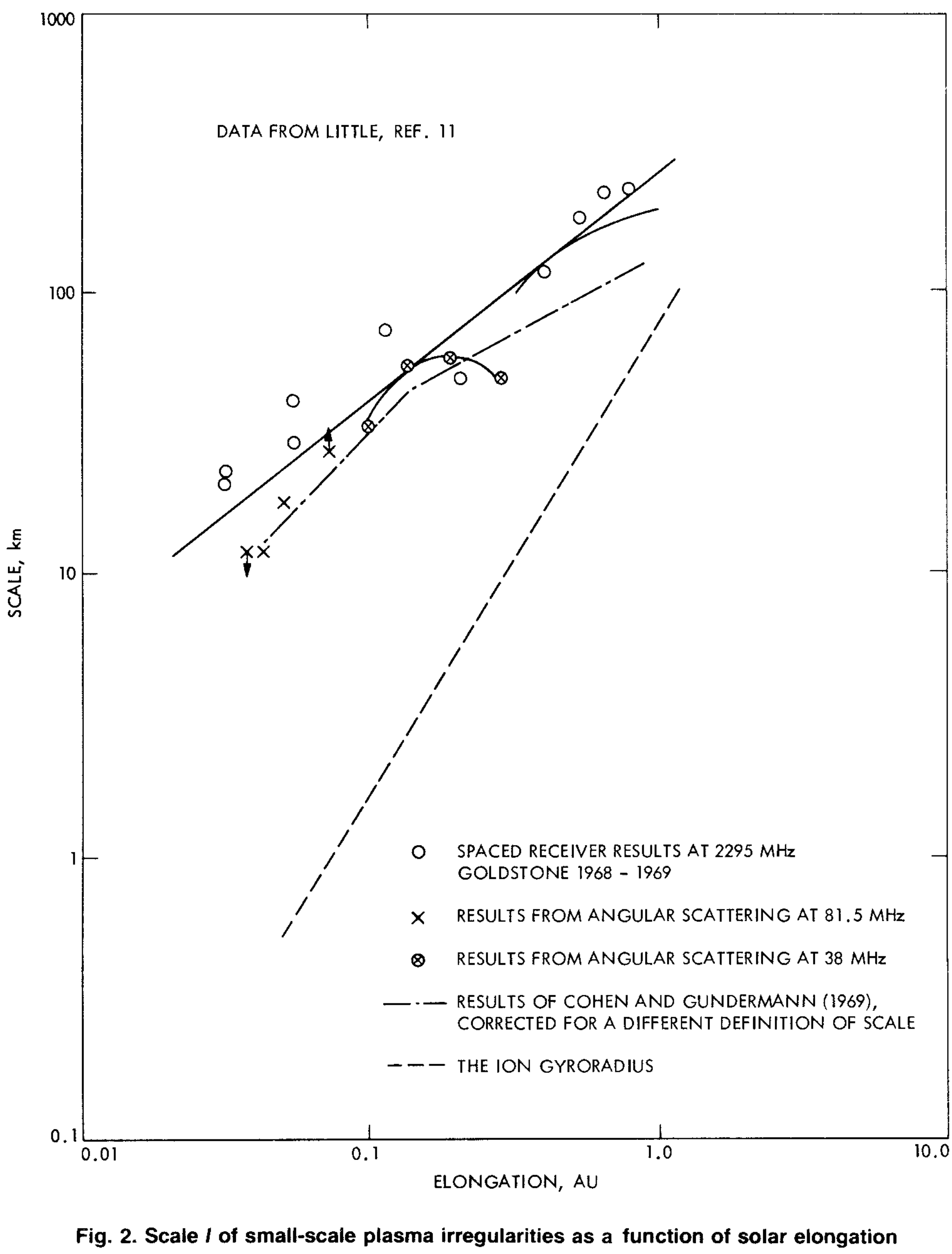

Fig. 2. Scale l of small-scale plasma irregularities as a function of solar elongation